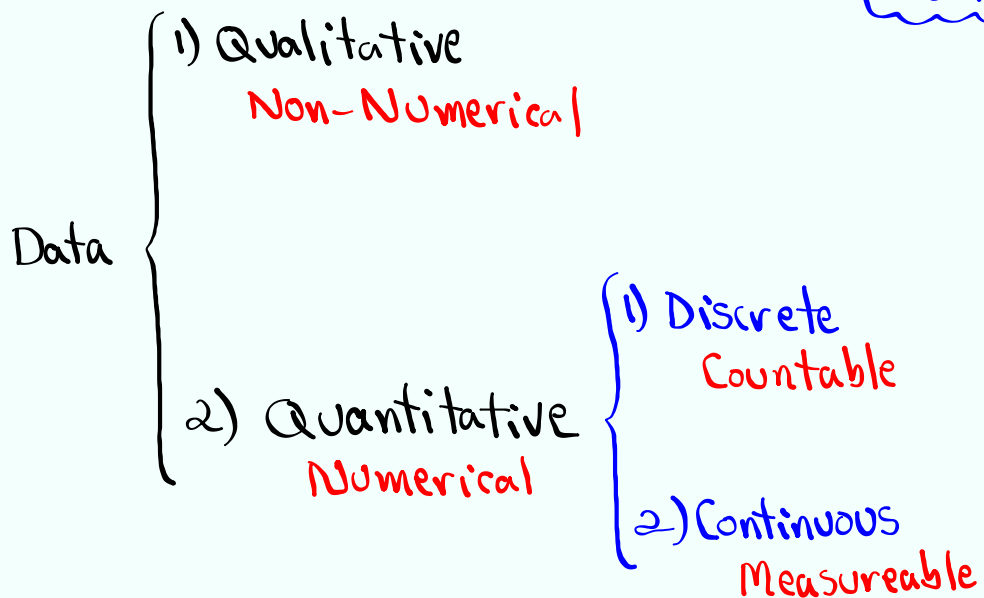


Statistics Lecture 8



Feb 19-8:47 AM



Oct 18-11:33 AM

Let x be a discrete random variable with Prob. dist. $P(x)$. \rightarrow Countable

Prob. dist. provides prob. of all possible outcomes.

Prob. dist. can be

- 1) in the form of a chart or table
- 2) in the form of a graph
- 3) in using some formula.
- 4) we could find it by concept of Prob.

Oct 18-11:35 AM

Some rules:

$$1) 0 \leq P(x) \leq 1$$

$$2) \sum P(x) = 1 \Rightarrow \text{Sum of all prob.} = 1$$

$$3) P(x) = 1 \Leftrightarrow \text{Sure event}$$

$$4) P(x) = 0 \Leftrightarrow \text{Impossible event}$$

$$5) 0 < P(x) \leq .05 \Leftrightarrow \text{Rare event}$$

Oct 18-11:40 AM

Consider the chart below 1) Verify $\sum P(x) = 1$

for discrete random variable $.2 + .5 + .3 = 1$

x with prob. dist. $P(x)$ 2) $P(x \leq 2) =$

$$.2 + .5 = .7$$

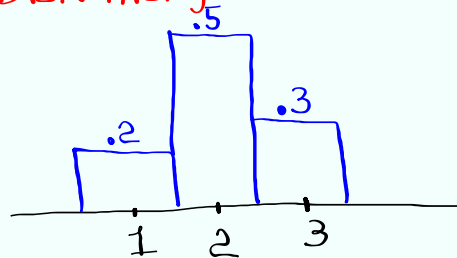
x	$P(x)$
1	.2
2	.5
3	.3

$$3) P(x \geq 2) = .5 + .3 = .8$$

4) Draw Prob. Dist. Histogram.

$x \rightarrow$ Midpoint

$P(x) \rightarrow$ Rel. F.



Oct 18-11:43 AM

Consider the chart below

for discrete random variable

x with prob. dist. $P(x)$: 1) Find $P(x=4)$

$$= 1 - [.15 + .25 + .4]$$

$$\uparrow$$

$$\text{Total Prob.} = .2$$

x	$P(x)$
1	.15
2	.25
3	.4
4	.2

2) Find $P(x=2 \text{ or } x=3)$

$$= .25 + .4 = .65$$

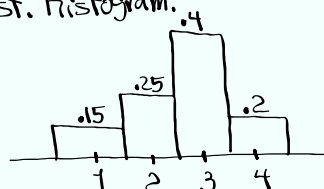
$$3) \text{ Find } P(x \geq 2) = 1 - .15 = .85$$

\uparrow
Total Prob.

4) Draw Prob. dist. histogram.

$x \rightarrow$ Midpoint

$P(x) \rightarrow$ Rel. F.



Oct 18-11:48 AM

A piggy bank has 2 Dimes & 3 Nickels
 Take 2 Coins, No replacement

Sample Space

$NN \rightarrow 10¢ \quad P(10¢) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = .3$
 $ND \rightarrow 15¢ \quad P(15¢) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20} = .6$
 $DN \rightarrow 15¢$
 $DD \rightarrow 20¢ \quad P(20¢) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = .1$

Total ¢	P(Total ¢)
10	.3
15	.6
20	.1

$P(\text{Total is } 10¢ \text{ or } 20¢) = .3 + .1 = \boxed{.4}$
 Prob. dist. histogram

Oct 18-11:56 AM

Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) Verify $\sum P(x) = 1$
 $.3 + .5 + .2 = 1 \checkmark$

2) $\sum xP(x) =$
 $.3 + 1.0 + .6 = 1.9$

3) $\sum x^2P(x) =$
 $.3 + 2.0 + 1.8 = 4.1$

4) Compute $\sum x^2P(x) - (\sum xP(x))^2$
 $= 4.1 - 1.9^2 = \boxed{.49}$

5) $\sqrt{\text{Last Ans.}} = \sqrt{.49} = \boxed{.7}$

Oct 18-12:04 PM

Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.1	.1	.1
2	.2	.4	.8
3	.4	1.2	3.6
4	.3	1.2	4.8

1) Verify $\sum P(x) = 1$
 $.1 + .2 + .4 + .3 = 1 \checkmark$

2) $\sum xP(x)$
 $\boxed{2.9}$

3) $\sum x^2P(x)$
 $\boxed{9.3}$

4) Compute $\sum x^2P(x) - (\sum xP(x))^2$
 $= 9.3 - 2.9^2 = \boxed{.89}$

5) $\sqrt{\text{last Ans.}} = \sqrt{.89} \approx \boxed{.943}$

6) $P(x \neq 1) = \underset{\substack{\uparrow \\ \text{Total} \\ \text{Prob.}}}{1} - P(x=1) = 1 - .1 = \boxed{.9}$

Oct 18-12:12 PM

4 Females, 6 Males, Select 3 people
 Let x be # of females

Sample Space:

- FFF $x=3$ $P(x=3) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{30}$
- FFM $x=2$ $P(x=2) = 3 \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8} = \frac{3}{10}$
- FMF $x=1$ $P(x=1) = 3 \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \frac{1}{2}$
- FMM $x=0$ $P(x=1) = 3 \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \frac{1}{2}$
- MFF $x=0$ $P(x=0) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{1}{6}$
- MFM
- MMF
- MMM

$\sum P(x) = 1$
 $\frac{1}{30} + \frac{3}{10} + \frac{1}{2} + \frac{1}{6} = 1 \checkmark$

Prob. dist. Histogram

Oct 18-12:21 PM

Mean μ (mu)

Variance σ^2 (Sigma²)

Standard dev. σ (Sigma)

$$\mu = \sum x p(x)$$

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

$$\sigma = \sqrt{\sigma^2}$$

From earlier example

x	P(x)
1	.2
2	.5
3	.3

$$\mu = \sum x p(x)$$

$$= 1(.2) + 2(.5) + 3(.3) = 2.1$$

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

$$= 1^2(.2) + 2^2(.5) + 3^2(.3) - 2.1^2$$

$$= .49$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{.49} = .7$$

Now using TI:

$x \rightarrow L1$, $P(x) \rightarrow L2$

Use 1-Var Stats

with L1 & L2

VARS 5: Statistics

4: σ_x x^2 Enter

$\mu = \bar{x} = 2.1$

$\sigma = \sigma_x = .7$

$n = 1 \leftarrow$ Total Prob.

$\sigma^2 = .49$

Oct 18-12:47 PM

From 2-Coin example

Total ϕ	P(Total ϕ)
10	.3
15	.6
20	.1

$$\mu = \bar{x} = 14$$

$$\sigma = \sigma_x = 3$$

$$n = 1$$

Total $\rightarrow L1$

P(Total) $\rightarrow L2$

Use 1-Var Stats

with L1 & L2

VARS 5: Statistics

4: σ_x x^2 Enter

$\sigma^2 = 9$

Oct 18-12:58 PM

From female example

# F	P(# F)
3	1/30
2	3/10
1	1/2
0	1/6

F → L1

P(# F) → L2

Use 1-Var Stats

with L1 & L2

$$\mu = \bar{x} = 1.2$$

$$\sigma = \sigma_x = .748$$

$$n = 1$$

VARS 5: Statistics

4: σ_x x^2 Math 1: \rightarrow Frac Enter

$$\sigma^2 = \frac{14}{25}$$

Oct 18-1:04 PM

Consider the chart below

x	P(x)
1	.2
2	.1
3	.2
4	.3
5	.2

1) $P(x=5)$

$$= 1 - (.2 + .1 + .2 + .3)$$

↑
Total Prob. = 1 - .8 = .2

2) $P(2 \leq x \leq 4)$

$$= .1 + .2 + .3 = .6$$

clear all lists

x → L1, P(x) → L2

3) Find μ & σ , Round up to whole #

1-Var Stats

with L1 & L2

$$\mu = \bar{x} = 3.2 \quad \mu = 4$$

$$\sigma = \sigma_x = 1.4 \quad \sigma = 2$$

Find σ^2 in reduced frac.

$$\sigma^2 = \frac{49}{25}$$

VARS 5: Statistics

4: σ_x x^2 Math 1: \rightarrow Frac Enter

68% Range

$$\mu \pm \sigma$$

$$= 4 \pm 2 \rightarrow 2 to 6$$

95% Range $\mu \pm 2\sigma$

$$= 4 \pm 2(2) \rightarrow 0 to 8$$

Oct 18-1:09 PM

Expected Value

I sold 25 TKTs @ \$10 each \Rightarrow \$250

I am giving away a Calc. \Rightarrow \$100

Net	P(Net)		Net Profit \$150
10 - 100	1/25	winning TKT	$\frac{\$150}{\$25} = \$6$
10 - 0	24/25	Losing TKTs	

Use (1-Var Stats) with L1 & L2

Net \rightarrow L1
P(Net) \rightarrow L2

E.V. = $\mu = \bar{x}$
\$6

Oct 18-1:20 PM

You are going on a trip.

You pay \$50 to insure your luggage

Any damages, Airline pays you \$1000

Prob. of any damage is 0.2% \rightarrow .002

Find expected value per policy sold.

Net	P(Net)		E.V. = $\mu = \bar{x}$
50 - 1000	.002	Damage	\$48
50 - 0	.998	No Damage	

Use (1-Var Stats) with L1 & L2

Net \rightarrow L1
P(Net) \rightarrow L2

Oct 18-1:27 PM

Pay me \$5
 Draw one card from a full-deck of playing cards.
 If you draw an Ace → I give \$50
 " " " a face → I give \$10
 Any other card → I give you nothing

Net	P(Net)	
5 - 50	4/52	Ace
5 - 10	12/52	Face
5 - 0	36/52	Any other card

Net → L1
 P(Net) → L2
 Use 1-Var Stats
 with L1 & L2
 Card E.V. = $\mu = \bar{x}$
 \$ -1.15

Oct 18-1:32 PM

Binomial Prob. dist. (SG 16)

- n independent events (Trials)
- Each trial has two outcomes.
 $P(\text{Success}) = p$ $P(\text{Failure}) = q$
 $p + q = 1$ $q = 1 - p$
 p & q remain unchanged for all trials.
- x → # of successes
 $n - x$ → # of failures

$$P(x) = {}^n C_x \cdot p^x \cdot q^{n-x}$$

↳ gives us how many ways we can have x successes.

Combination
 ${}^n C_r$ n choose r

${}^5 C_2 = 10$ 5 [math] [PRB] [nCr] [=] [enter]

${}^{12} C_5 = 792$ 12 [math] [PRB] [nCr] 5 [enter]

${}^{50} C_5 = 2,118,760$ 50 [math] [PRB] [nCr] 5 [enter]

Oct 18-1:51 PM

Consider a binomial Prob. dist with
 $n = 10$ and $p = .6$

$$\begin{aligned}
 1) q &= 1 - p = 1 - .6 = \boxed{.4} & 2) np &= 10(.6) = \boxed{6} \\
 3) npq &= 10(.6)(.4) = \boxed{2.4} & 4) \sqrt{npq} &= \sqrt{2.4} \approx \boxed{1.549} \\
 5) P(x=7) &= {}^{10}C_7 \cdot (.6)^7 \cdot (.4)^3 \approx \boxed{.215} \\
 n^C_x \cdot p^x \cdot q^{n-x} & \quad \hat{=} \quad \rightarrow
 \end{aligned}$$

Oct 18-2:01 PM

Consider a binomial Prob. dist with
 $n = 20$ and $p = .5$

$$\begin{aligned}
 1) q &= 1 - p = 1 - .5 = \boxed{.5} & 2) np &= 20(.5) = \boxed{10} \\
 3) npq &= 20(.5)(.5) = \boxed{5} & 4) \sqrt{npq} &= \sqrt{5} \approx \boxed{2} \\
 & & & \text{Round to whole \#} \\
 5) P(\text{exactly 12 Successes}) & \\
 P(x=12) &= {}^{20}C_{12} \cdot (.5)^{12} \cdot (.5)^8 = \boxed{.120} \\
 n^C_x \cdot p^x \cdot q^{n-x} &
 \end{aligned}$$

using TI Command

$\boxed{2nd}$ \boxed{VARS} \boxed{d} $\boxed{\text{Binom pdf}}$

Trials: 20
 $P: .5$
 $x\text{-Value: } 12$
 $\boxed{\text{Paste}} \boxed{\text{Enter}}$

} 20, .5, 12
 $\boxed{\text{enter}}$

Oct 18-2:07 PM

I flipped a loaded coin 100 times

$$P(\text{Tails}) = .7$$

$P(\text{exactly } 75 \text{ tails})$

$$x = 75$$

$$P(x = 75) = \text{binompdf}(100, .7, 75) = \boxed{.050}$$

$P(\text{at most } 75 \text{ tails})$

$$x \leq 75$$

$$P(x \leq 75) = \text{binomcdf}(100, .7, 75) = \boxed{.886}$$

Oct 18-2:17 PM

You are taking a True-False exam with 400 questions

Success is to correctly guess.

You are making random guess.

$$n = 400 \quad P = .5$$

$P(\text{guess correctly exactly } 200)$

$$x = 200$$

$$P(x = 200) = \text{binompdf}(400, .5, 200) = \boxed{.0398}$$

$P(\text{at most } 200 \text{ correct ans}) = \boxed{.520}$

$$x \leq 200$$

$$P(x \leq 200) = \text{binomcdf}(400, .5, 200)$$

Oct 18-2:25 PM

Class Quiz 4

x	$P(x)$
1	.20
2	.15
3	.25
4	.35
5	.05

Find

1) $\mu = 2.9 \approx \boxed{3}$

2) $\sigma = 1.221 \approx \boxed{1}$

3) $\sigma^2 = \boxed{\frac{149}{100}}$

} Round to
whole #} Reduced
fraction

Oct 18-2:31 PM